

Find the radius (and interval) of convergence for

Review

1. Use the function  $f(x) = \cos x$  to form the Maclaurin series

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

(a) Find the derivatives

$$f'(x) = -\sin x$$

$$f''(x) = -\cos x$$

$$f'''(x) = \sin x$$

$$f^{(4)}(x) = \cos x$$

(b) evaluate the derivatives at zero

$$f'(0) = 0$$

$$f''(0) = -1$$

$$f'''(0) = 0$$

$$f^{(4)}(0) = 1$$

(c) Assemble the series

$$\cos x = f(0) + \frac{f'(0)x^1}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{(4)}(0)x^4}{4!} + \dots$$

$$\cos x = 1 + \frac{0x}{1} + \frac{(-1)x^2}{2!} + 0 + \frac{(1)x^4}{4!} + \dots$$

(d) Use the ratio test to conclude that the series converges for all  $x$

$$\lim_{n \rightarrow \infty} \left| \frac{x^{2n+1}}{(2n+1)(2n)!} \cdot \frac{(2n)!}{x^{2n}} \right| = \lim_{n \rightarrow \infty} \left| \frac{x}{2n+1} \right| = 0 < 1 \quad (-\infty, \infty)$$

